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Continuation of entropy

Perron theory in FOS

Paper by Newhouse in 80s

Rem h as a function of map, measure, both.

USC of entropy from smoothness

upper semi continuity wh for C^∞ (Yomdin)
or for uniform hyperbolicity
false for $C^r, r < \infty$ (non hyperbolic)

- C^r perturbation of homoclinic intervals
→ h_{top} (small horseshoe) can approach min exponent
differentiability
Conclusion failure of USC of h_{top} in $C^r (r < \infty)$.

- Similar for h_{top} : not USC in C^r
(for instance, Young: $h_{\text{top}}(2\text{-d flow}) = 0$)

USC of entropy from expansiveness

Bowen's (asymptotic) h -expansiveness.

$$\begin{aligned} & \exists \varepsilon_0 > 0 \\ & \hookrightarrow h_{\text{top}}(B_{\varepsilon_0}(x)) = 0 \\ & \hookrightarrow \lim_{\varepsilon \rightarrow 0} \sup_x h_{\text{top}}(B_\varepsilon(x)) = 0. \end{aligned}$$

Rem implies USC of metric entropy
for top entropy, need local uniformity.

Rem pw affine surface diffeol (recent (3 years ago))
counter example by student of Newhouse

Def

f is ε -entropy expansive if $h_\varepsilon^*(f) = \sup_x h_{top}(f|_{B_\varepsilon(x)}) = 0$

f is ε - C^r robustly entropy expansive if \exists a neighborhood of f in C^r topology where $h_\varepsilon^*(f) = 0$

f is asymptotically entropy expansive if ...

f is robustly ... if the previous convergence holds uniformly on a neighborhood of f .

Bowen: $h_\mu(f, A) \leq h_\mu(f) \leq h_\mu(f, A) + h_\varepsilon^*(f)$
if $\text{diam } A \leq \varepsilon$.

(this shows USC under expansivity properties).

Thm (Liao, Viana, J. Yang)

in $\text{Diff}^2(M)$: away from tangencies, robust entropy expansiveness holds.

Corollaries

1) h_{top} is USC on $\text{Diff}^2(M) \setminus \overline{HT^{-1}(M)}$

2) Shub's conjecture away from tangencies

$$h_{top}(f) \geq \log \rho(f_*)$$

\uparrow total action on homology

asked for Lipschitz but false



($0 \mapsto 2\theta$)
(ρ goes to 0)

Yom din C^∞ ok

stands for C^1

Xia, Sagun for PH $d_c = 1$

proof: $\rho(f_*)$ is locally constant.
 $f_n \xrightarrow{C^1} f^*$

$$h_{\text{top}}(f_n) \geq \rho(f_n) = \rho(f_*)$$

usc yields the conclusion.

papers by Newhouse-Young, Young & Crosser, Pacific, Day-Fisher - Viete

Proof of the theorem

① Lemma

f is entropy-expansive iff $\forall \mu \in \text{Vol}_{\text{me}}(f)$
for μ -a.e x $h_{\varepsilon}^*(f, B_{\infty}(x, \varepsilon)) = 0$

Rem a more general statement is true
(replace 0 by any number).

② (Crosser, χ) on the support of any
ergodic inv proba, there is a PH
splitting (with $d_c \leq 1$).
[away from tangencies]

based on Mañé's ergodic lemma & Wen: far
from tangencies, domination on periodic points.

↳ for center-bundle
splitting into
1-d (with
domination)
this holds for
open & dense set
away from
tangencies
(Crosser,
Samburov,
Dawae Young)

Metric entropy of volume preserving diffeos

thm h_{vol} is continuous on a generic subset of $\text{Diff}^1(M)$

Cor Pesin formula for C^1 diffeos.

Cor $\lambda_1(f)$ is generically continuous in $C^1(\text{vol})$

Bodhi Viana: For generic f ,
 on a.e. x there is a dominated splitting corresponding
 to the exponents.

→ $B_\infty(x, \varepsilon_x)$ is contained in a m fold tangent
 to EC for some $\varepsilon_x > 0$ depending on x

→ $\text{hyp}_p(B_\infty(x, \varepsilon_x))$ is small (ε_x reduced if
 necessary)

Gives Λ_f with $\text{Leb}(\Lambda_f) \approx 1$ where the
 above holds with some uniformity.

Aula-Bodhi: (Observation)

$$c > 0 \quad \lambda < 1$$

$$\forall f \in \text{rang}_{\text{Vol}}^{-1}(M)$$

$$\Lambda_f := \{x : \exists \text{ d.s. on orb}(x)\}$$

uniform

$$f_n \xrightarrow{r_1} f$$

$$\overline{\text{lim}} \Lambda_{f_n} \subset \Lambda_f$$

$$\Rightarrow \overline{\text{lim}} \text{Leb}(\Lambda_{f_n}) \leq \text{Leb}(\Lambda_f)$$

semi-continuity \Rightarrow continuous on generic
 subset.

$$\left. \begin{array}{l} \text{where } \Pi \| Df |_{EC^s(f^k_x)} \| \leq CA^k \\ \Pi \| Df' |_{E^{cu}(f^{-k}_x)} \| \leq CA^k \end{array} \right\}$$

$$\forall n \geq 0$$

by
 uniformity

Applications

1) C^2 transitive diffeos: with unique SRB μ_f .
 we have stochastic stability in C^2 topology.
 (weak star)

this extends to $C^2 f_n \xrightarrow{C^2} f \in C^2$

uses thermodynamical formalism & uniqueness

Proof

$$P(\mu_n) = 0$$

Consider an accumulation point μ_0 in weak*

$$\text{usc} \Rightarrow P(\mu_0) \geq 0 \Rightarrow P(\mu_0) = 0$$

$$\Rightarrow \mu_0 \notin B.$$

2) C^1 generically transitive Anosov
(see also Hao & Xu) quas on S^1 .

Keller: $\#\{\mu : P_S(\mu) \geq 0\} = 1$ then this is ~~set~~ physical (unique)
(U)

For $f \in C^2$, (U) is known.

For $f \in C^1$, $P(f) \geq 0$

Consider $E: f \mapsto E_f := \#\{\mu : P(\mu) \geq 0\}$

E is upper semi continuous, therefore continuous in the Hausdorff topology.

$\#E_f = 1$ if C^2 (or dense)

Therefore $\#E_f = 1$, i.e. (U), holds on a generic subset.

Mostly contracting diffeos C^2

$$E^{CS} \oplus E^U$$

(def of Bonatti-Viana)

~~any D_0 with positive volume~~
any disk in F^u contains a set of positive volume

\Leftrightarrow For any m -Gibbs μ $\uparrow^{CS}(\mu) < 0$ $\left| \lim_{n \rightarrow \infty} \frac{1}{n} \log \|Df^n\|_{CS}^m \right|_0 < 0$

BV \Rightarrow finitely many physical neurons \otimes
 \star V basis $= M \bmod 0$.

- the condition "mostly contracting" is C^2 open
(M. Anderson)

• It is actually C^1 open (among C^2)

\hookrightarrow PH, $d_c = 1$ $\textcircled{1}$

$\textcircled{2}$ Mostly contracting

$\textcircled{3}$ $\forall \mu \in E_f$ lines $\lambda^{cs}(\mu) \neq 0$

\leftarrow satisfied
by Mañé's example

$\textcircled{1} \cap \textcircled{2} \cap \textcircled{3}$ is C^1 open

\uparrow most delicate. Gives $u \in E_f$.

Moreover, $\textcircled{1}$ extends to a generic subset C^1 .